# On Formation of Disk Galaxies from Spherical Primordial Fluctuations

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**Abstract.** The spherical gravitational collapse and virialization of arbitrary density fluctuations in an expanding universe is studied. In the context of the standard cosmological model and the peak *ansatz*, disk galaxies are supposed to be the product of "extended" gravitational collapses of regions surrounding local-maxima of the density fluctuation field. The substructure over the main profile (merging) is considered as a second-order phenomenon. The range and distribution of possible mass growth histories or accretion regimes for a given present-day mass is estimated and used as the initial condition for galaxy formation. Considering the effect of dark halo contraction due to disk formation, the final rotation curves are calculated. If the structural properties of dark halos are important in establishing the Hubble sequence's properties of visible galaxies, then this sequence is defined not only by the mass, but among other possible parameters, by the accretion regime.

## 1. Introduction

The Alpine mountains here in Sesto are a vivid representation of what could have been the primordial density fluctuation field: peaks, mountainsides, valleys; some distributed in a superposed fashion, others isolated and so on. If we accept the gravitational paradigm (for alternatives see Babul *et al*. 1994) and the peak density *ansatz* then the cosmic structures that one observes today should have emerged from the peaks of this initial density fluctuation field. Although some N-body experiments have shown that the peaks can be disrupted before collapsing it is still precipitous to make conclusions when in fact one is using inaccurate peak statistics (Manrique & Salvador-Sole 1996 and references therein). Anyway, in the case of disk galaxies the peak *ansatz* is more suitable because they are typically located in less dense environments where the shear caused by the surrounding fluctuations is less important. The peaks from which they emerge, not being enhanced by larger scale fluctuations, should have been high, therefore the gravitational potential of the matter collapsing around them would tend to be spherically symmetric. From the gravitational collapse of the regions around peaks arise the dark matter halos, where subsequently the baryon matter cools and falls to the center forming the visible galaxies. A

link (if any) between the present-day observable properties of galaxies and the initial conditions can be established only by taking into account the intermediate processes of the formation and evolution of galaxies. Our presentations, this paper and Firmani *et al.*1996 (FAH), go in this direction.

## 2. Spherical gravitational collapse

The spherical gravitational collapse model (Gunn & Gott 1972, Peebles 1980) allows us to calculate the imaginary structure one would obtain by freezing all collapsing shells at their turn around radius:

$$\rho_m(r_m) = \frac{9\pi^2}{16(1+\alpha)}\frac{1}{6\pi G t_m^2} \propto \frac{r_m^{-\frac{3\alpha}{1+\alpha}}}{1+\alpha} \tag{1}$$

where $\alpha(x) \equiv -\frac{d\ln\Delta_0(x)}{d\ln x}$ is the local slope of the cumulative density profile $\Delta_0(x) \propto x^{-\alpha(x)}$, and the proportionality in eq. 1 is valid for self-similar collapse. If the final virialized radius $r_v$ scales with $r_m$, that is $r_v = Fr_m$ with $F = const$ for a given $\alpha$, then the density profile of the halo will be

$$\rho_v(r_v) \propto \frac{r_v^{-\frac{3\alpha}{1+\alpha}}}{1+\alpha} \tag{2}$$

Because our approach will tend to tune the present-day disk galaxy properties to their initial conditions ($FAH$), it is more convenient to express these initial conditions through the mass growth history ($MGH$) back in time, rather than through an initial density profile (both are equivalent from a conceptual point of view). Here we parametrize the $MGH$ by a power-law:

$$M(t) = M_0\left(\frac{t}{t_0}\right)^\gamma \tag{3}$$

where $\gamma(t)$, the accretion parameter, is constant in the case of self-similar collapse, and $M_0$ is the mass at the present epoch $t_0$.

It is straighforward to calculate eq. 2 for this case:

$$\rho_v(r_v) \simeq 6\cdot 10^5\left(\frac{F}{0.5}\right)^2 \rho_0 r_v^{-\frac{6}{2+\gamma}} \tag{4}$$

where $F$ is the collapse factor and $\rho_0$ is the present-day critical density (only an Einstein-de Sitter model is considered here, and $H_0 = 50Kms^{-1}Mpc^{-1}$). The rotation curve will be

$$V(r_v) \simeq 27.3\left(\frac{F}{0.5}\right)^{-1/2} \frac{M_{10}^{\frac{1}{2+\gamma}}}{\left[58\left(\frac{0.5}{F}\right)\right]^{\frac{\gamma-1}{2+\gamma}}} r_v^{-\frac{\gamma-1}{2+\gamma}} Km/s \tag{5}$$

where $M_{10}$ is $M_0$ in units of $10^{10}M_\odot$. Some interesting features can be readily seen from these simple analytical results. When $\gamma \approx 1$ (the high accretion

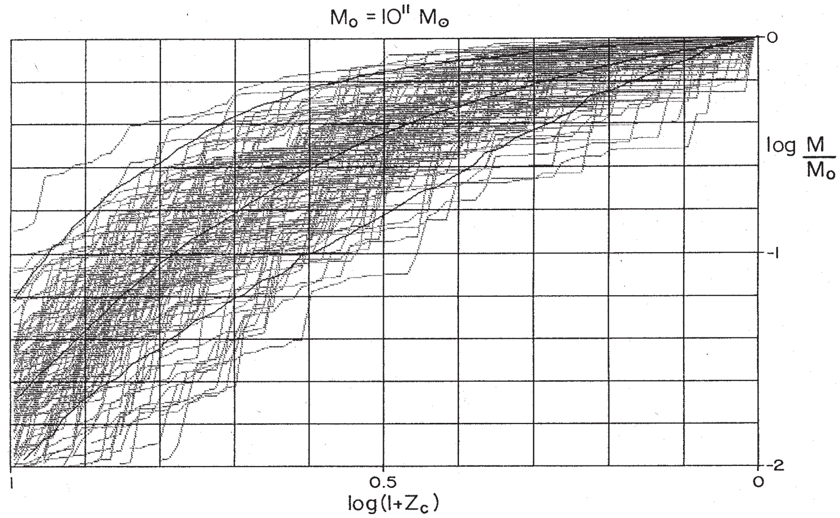


Figure 1. Statistical realizations of possible mass growth histories for a $10^{11}M_{\odot}$ dark halo. From top to bottom the low, average, and high accretion regimes are plotted (see text).

regime) the asymptotic rotation curve is flat, while if $\gamma \gtrsim 0$(the low accretion regime) the halo is a very concentrated structure.

To study the virialization in more general cases (non scale-invariant $MGHs$, nonradial orbits and $F(r) \neq const$) the shell-crossing phases should be calculated. The crucial assumption in the simplification of the problem is to consider the orbital period of the inner shell much less than the time-scale of accretion of outer shells. In such case the dynamics of the inner shell admits an adiabatic invariant, that for radial orbits yields

$$M(< r_{ap}) \propto r_{ap}^{-1} \tag{6}$$

(Gunn 1977). $r_{ap}$ is the apapsis radius, where a particle spends most of its orbital time. It is just the secular contraction of $r_{ap}$, by the penetration of outer shells, that should be calculated . The procedure of Gunn 1977 and Filmore & Goldreich 1984 consists of evaluating the change $F^{-1}$of the enclosed mass within a given shell due to the infall of outer shells, and then one uses relation (6). The mass contained within a given apapsis $r_p$ consists of two contributions: the mass of all particles with apapsis radius less than $r_{ap}$, and the mass of outer infalling particles which spend some fraction of their orbital time within $r_{ap}$. The evaluation of the second contribution is the crucial step (Zaroubi & Hoffman 1993). We use a statistical approach to the problem, and generalize the cases of Zaroubi & Hoffman for nonradial orbits and arbitrary profiles ($MGHs$ in our case). An iterative procedure is used to find $F^{-1}$. Typically the convergence is fast. Thus, given the $MGH$, we calculate semi-analytically the structure of the collapsed configuration. The degree of orbital nonradiality measured through

$E \equiv \frac{\min or-axis}{mayor-axis}$, is a free parameter which affects the central regions of the virialized halo. If $E << 1$ (radial case) the halo tends to be coreless (see the rotation curves in Fig.5 )

## 3. The mass growth histories

The statistical properties of the primordial density fluctuation field depend on the physical mechanism in the very early universe that generated the fluctuations. It is widely accepted (and as a *first* approximation seems to work) possibility the inflation produced a Gaussian random field with a near scale-invariant power spectrum. Since the processed power spectrum after considering the damping and stagnation processes is such that small scales have more density contrast, the evolution of fluctuations will be hierarchical, from small to large scales. Some statistics were worked out for this general case to predict the development of the cosmic structure. An interesting step was the estimation of the conditional probability of finding a collapsed object of mass $M_2$ at time $z_2$ provided it is embedded in a larger collapsed object $M_1$ at an earlier time $z_1$ (Bower 1991, Bond *et al*. 1991, Lacey & Cole 1993). This allows one to roughly follow the $MGHs$ back in time for a fixed mass at the present epoch. Note that because the mass of the given object is forced to collapse today, we are already selecting a special (isolated) region of the field. As was pointed-out in the introduction, disk galaxies probably emerged from regions of the field where the signal greatly dominates the noise, that is, their $MGHs$ are nearer to a coherent accretion process, rather than a chaotic and discontinuous aggregation of collapsed lumps.

As in Lacey & Cole we generate Monte Carlo trajectories using the mentioned probability distribution, and maintaining at every step the individuality of the progenitor. The average of these special trajectories for a given present-day mass is calculated and used as an initial condition of galaxy formation. However, other evolutionary trajectories are possible, covering a great variety of disk galaxies with a given mass (the Hubble sequence). Thus, we estimate the two, still significant, extrema by finding the deviations from the most probable trajectory which constitutes roughly 10% of all the realizations and averaging them. Since the $MGHs$ are fixed to the present mass, these extreme trajectories reflect the range of possible accretion regimes of disk galaxies which for galactic masses oscillates between $\gamma \sim 0.2$ (low accretion regime) and $\gamma \sim 1$ (high accretion regime). The average trajectory also slowly varies with mass ($\gamma$ rises with the mass). We used a standard cold dark matter model with power spectrum normalized to $COBE$ (Stompor *et al*. 1995). Fig. 1 shows a sample of 1000 halo histories (only 1 of every 10 trajectories are plotted) for a dark halo of $10^{11} M_{\odot}$ and the low, average, and high accretion regime's cases.

## 4. Results

Figure 2 presents the rotation curves and density profiles of a $10^{12} M_{\odot}$ halo corresponding to the low, average, and high accretion rates. It is seen clearly that halos formed by fast collapse are more concentrated than those formed by

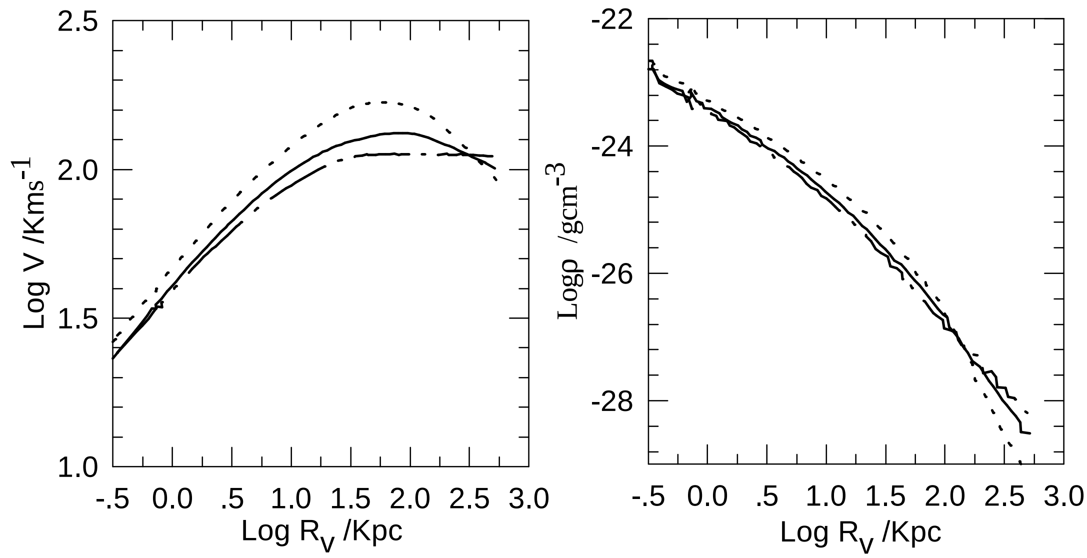


Figure 2. The circular velocities and density profiles of a $10^{12} M_{\odot}$ dark halo. Dotted, solid, and dot.dashed lines correspond to the low, average, and high accretion regimes, respectively

an extended collapse. In Figure 3 the rotation curves for the average trajectory of $10^{10} M_{\odot}$, $10^{11} M_{\odot}$, and $10^{12} M_{\odot}$ halos are plotted. Low mass systems turn out to be a little more concentrated than high mass ones (see also Navarro *et al.* 1996 who calculated the structure of dark halos using N-body simulations). The combination of these results suggests that if the structural properties of dark halos are important in establishing the Hubble sequence's properties of visible galaxies, then this sequence is defined not only by one parameter, *i.e* the mass, but at least by one more, the accretion regime. In $FAH$ we show that in fact the accretion regime is also important in defining the star formation histories of galactic disks.

The relation between mass and maximum circular velocity (the "cosmological" Tully-Fisher relation $T-F$) we obtain from the average trajectories is $M \propto V^{3.1}$. Properly accounting for the deviations around these trajectories the *r.m.s.* scatter in the cosmological $T-F$ drops slowly for high mass halos, with a rough average of $\Delta V/V \simeq 0.25$ corresponding to $\sigma \simeq 0.8 mag$. This is a high value compared with the observational $T-F$ scatter. The evolutionary models presented in $FAH$ give a scatter in B luminosity of $\sim 0.5 mag$. Some mechanism intrinsic to galactic evolution or related to the formation of the disk could have reduced this scatter.

In order to obtain rotation curves resembling those of the observed galaxies, it is necessary to consider the further dissipative collapse of baryonic matter and its gravitational pull on the dark halo. This effect was calculated employing a simple analytical model (Flores *et al.* 1993) based on the adiabatic invariant formalism (see Firmani *et al.* 1996). The disk is built up as explained in $FAH$. The effect is considerable in the central halo regions. Figure 4 shows the rotation

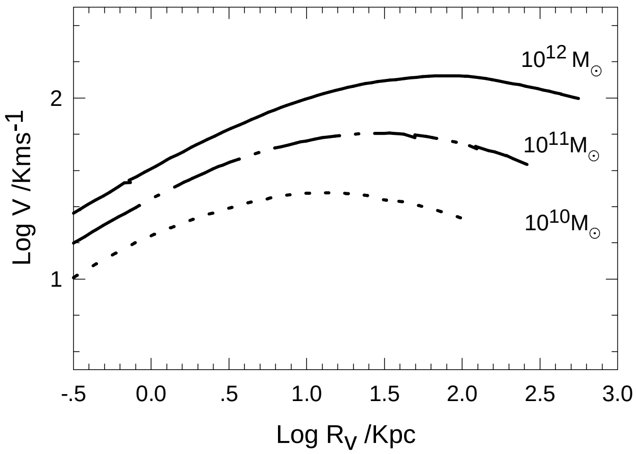


Figure 3. The circular velocities of dark halos of $10^{10}M_\odot$, $10^{11}M_\odot$, and $10^{12}M_\odot$ in the case of average accretion regime. Low mass halos reach their maximum circular velocity at smaller radii w.r.t. the total virial radius than higher mass halos

curve decomposition for a $10^{12}M_\odot$ galaxy. The halo rotation curves before and after the disk formation are plotted. The galactic rotation curves in all cases are nearly flat (see also figure 3 in $FAH$). However, going into more detail, for the range of masses analized here ($10^{10}M_\odot$-$10^{12}M_\odot$) the rotation curves within the optical radius show a very slow variation in shape, from gently rising in low mass galaxies to declining in more massive galaxies. Based on a large data set Persic & Salucci 1995, and Persic *et al*. 1996 have established a similar trend, but more pronounced than the results of our models.

## 5. Discussion and conclusions

In section 2 it was mentioned that the degree of nonradiality $E$ is a free parameter in our calculations. $E$ was fixed in such a way that the galactic halos (1) have a core, and (2) the disk rotation curve be of the order of the halo rotation curve. It is well known that dwarf galaxies are dominated by dark matter even in their central regions. So their rotation curves are a good tracer of the dark halo. As Moore 1994 and Flores *et al*. 1995 pointed out, the observed rotation curves of dwarf galaxies show the existence of a pronounced core. On the other hand, standard rotation curve decompositions give a dominant contribution of the disk over that of the dark halo in the center of high luminosity (normal) galaxies. For comparison we reconstructed the rotation curves obtained by N-body simulations in Navarro *et al*. 1996 using their fitting formula (a $COBE$ normalization is used here). Figure 5 shows the comparison of these rotation curves with the one obtained here. If we fix $E$ to lower values the agreement is good, although the observational criteria (1) and (2) will not be satisfied. This discrepancy is atennuated for low-$\Omega$ CDM models (Navarro 1996). It could be also suggesting a more critical analysis of the underpinning of the structure formation and evo-

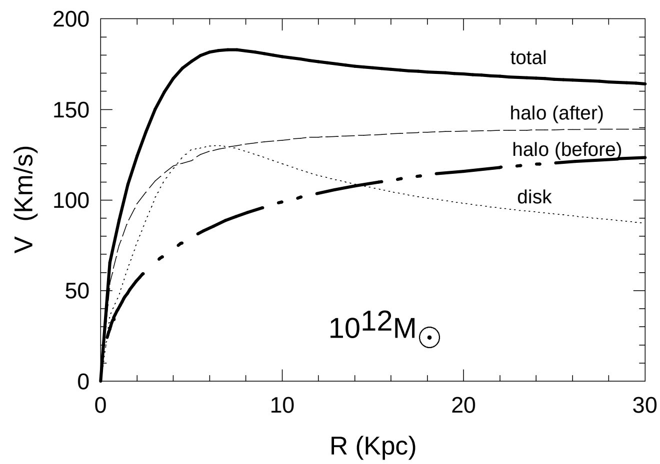


Figure 4. Rotation curve decomposition for a $10^{12}M_{\odot}$ galaxy and for the average accretion regime case

lution theory, namely the random-phases hypothesis. Small scales, due to the shape of the power spectrum, are more sensitive to this hypothesis.

It is important to note that Persic *et al*. 1996 have found that the degree of dominance of disk over halo in the central regions of galaxies is an increasing monotonic function of luminosity. This could be related to the result mentioned above that low mass dark halos are systematically more concentrated than their high mass counterparts. On the other hand, it was stated that for every mass, due to the different possible $MGHs$, there is a sequence of dark structures — a factor which could introduce a large dispersion in the relation between the galactic mass (but still not the disk luminosity) and the disk/halo mass ratio. This question will be treated in detail in a future paper.

Throughout the present work it was demonstrated that the amplitude and shape of galactic rotation curves for a given mass and cosmological model depends on the accretion regime, the level of substructure around the peaks, and the further collapse of baryonic matter. The different accretion regimes generated from initial cosmological conditions establish a range of structural halo properties that could be important in understanding the origin of the Hubble sequence.

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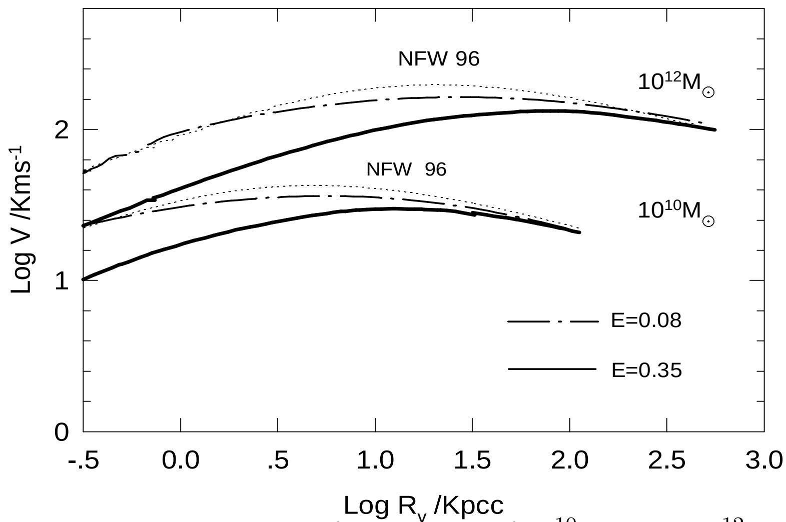


Figure 5. Rotation curves of dark halos of $10^{10}M_\odot$ and $10^{12}M_\odot$ for the average accretion regime case and for two different degrees of orbital nonradiality (shown in the figure). The dotted lines correspond to the fitting formula of N-body simulations (Navarro *et al.* 1996)


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